❖ CS 598: Computational Topology (Fall 2009) ❖

❖ About ❖ Schedule ❖ References ❖ Coursework ❖ Projects ❖

Schedule

Future dates are approximate, and future lecture topics are subject to change. (In particular, some topics slated for a single lecture may take more than one; others may be dropped due to lack of time.) Some keywords are links to relevant articles in Wikipedia. Key references for each lecture are highlighted. Other references contain related material (such as historical references or later improvements and extensions) that I may cover in class only briefly or not at all.

Lecture notes are available for some topics—look for links in the left margin. Most of these notes are sketchy; most (if not all) of them are also buggy. Feedback is welcome, especially if you find mistakes!

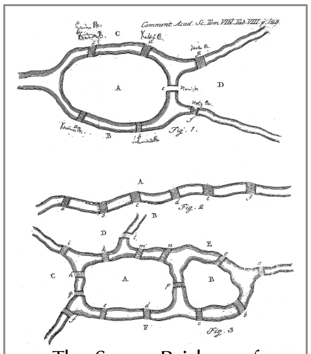
❖ Introduction ❖ Paths in the plane ❖ Curves on surfaces ❖
❖ Topological graph theory ❖ Normal curves and surfaces ❖
Homology ❖

❖ This week's lecture ❖ Future Topics ❖ Related Topics ❖

❖ Introduction ❖

Tue, Aug 25
~~no notes~~

Introduction, history, overview, and administrivia.



The Seven Bridges of Königsberg and two other bridge systems. From Euler [Commentarii 1735]

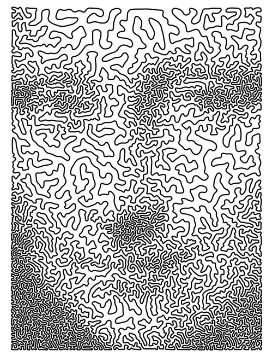
- Leonhard Euler. Solutio problematis ad geometriam situs pertinentis. *Commentarii academiae scientiarum Petropolitanae* 8:128–140, 1741. Presented to the St. Petersburg Academy on August 26, 1735. Reprinted in *Opera Omnia* 1(7):1–10. *[The famous Königsberg bridge paper, arguably the first published result in topology. Only proves that a graph with more than two odd vertices has no Euler tour. The paper also **very** briefly sketches the first topological algorithm—computing an Euler tour in a graph with at most two odd vertices.]*
- Carl Hierholzer. Über die Möglichkeit, einen Linienzug Ohne Wiederholung und ohne Unterbrechnung zu umfahren. *Mathematische Annalen* 6:30–32, 1873. *[The first **proof** of that a graph with at most two odd vertices has an Euler tour, given in the form of an algorithm. Hierholzer attributes the main ideas to Listing's 1847 treatise Volesung zur Topologie.]*

- Henri Poincaré. Analysis situs. *Journal de l'École Polytechnique* 1:1–121, 1895. Reprinted in *Ouvres* VI:193–288. *[The beginnings of topology in its modern form.]*
- Henri Poincaré. Second complément à l'analysis situs. *Proceedings of the London Mathematical Society* 32:277–308, 1900. Reprinted in *Ouvres* VI:338–370. *[Includes, among many other results, the standard algorithm for computing the Betti numbers of a simplicial complex, via elementary row and column operations, starting with the boundary matrices. Poincaré was apparently unaware of the equivalent algorithm published by Smith in 1861 to compute the Smith normal form of an integer matrix.]*
- Wolfgang Haken. Theorie der Normalflächen: Ein Isotopiekriterium für den Kreisknoten. *Acta Mathematica* 105:245–375, 1961. *[Haken's quadruply-exponential-time algorithm to recognizing whether a given knot is trivial.]*

❖ Paths in the plane ❖

Thu, Aug 27 [notes]

The Jordan-Schönflies Theorem, testing whether a point lies inside a polygon



A simple polygon.
from Kaplan and Bosch
[BRIDGES 2005]

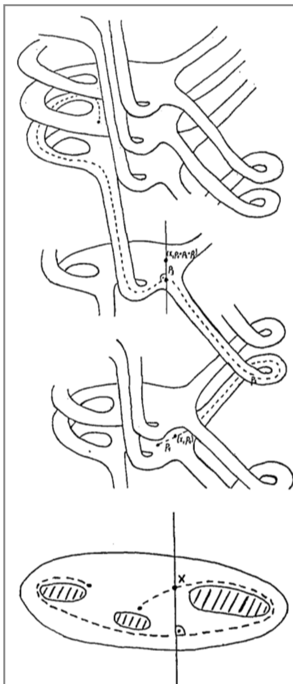
- W. Randolph Franklin. PNPOLY - Point Inclusion in Polygon Test. ⟨http://www.ecse.rpi.edu/Homepages/wrf/Research/Short_Notes/pnpoly.html⟩ , May 20, 2008. *[A particularly concise implementation of the ray-parity algorithm.]*
- Richard Hacker. Certification of Algorithm 112: Position of point relative to polygon. *Communications of the ACM* 5(12):606, 1962. *[Corrects a minor bug in Shimrat's algorithm.]*
- Camille Jordan. Courbes continues. *Cours d'Analyse de l'École Polytechnique*, 587–594, 1887. *[The first formal statement and proof of the theorem.]*
- Andrew Ranicki. Jordan curve theorem. ⟨http://www.maths.ed.ac.uk/~aar/jordan/⟩ , February 21, 2008. *[Pointers to several historical and canonical proofs.]*
- Arthur Schönflies. Beiträge zur Theorie der Punktmengen. III. *Mathematische Annallen* 62:286–328, 1906. *[First statement and proof of the Jordan-Schönflied theorem.]*
- M. Shimrat. Algorithm 112: Position of point relative to polygon. *Communications of the ACM* 5(8):434, 1962. *[The first publication of the ray-parity algorithm qua algorithm, but almost certainly not the first discovery.]*
- Oswald Veblen. Theory on plane curves in non-metrical analysis situs. *Transactions of the American Mathematical Society* 6:83–98, 1905. *[The first univerally-accepted proof of the theorem.]*

Tue, Sep 1 [notes]

Path homotopy, contractible, simply connected, covering space, universal cover

Shortest homotopic paths: triangulation, crossing sequences, reduction, sleeve, funnel

- Bernard Chazelle. [A theorem on polygon cutting with applications](). *Proceedings of the 23rd Annual IEEE Symposium on Foundations of Computer Science, 339–349, 1982.* *[First description of the funnel algorithm for shortest paths in sleeves; see section 5.1. The main result of the paper is algorithm to find a diagonal that splits any n-gon into two smaller polygons, each with at most 2n/3 vertices, in O(n) time. Applying this algorithm recursively builds a triangulation of the polygon in O(n log n) time.]*

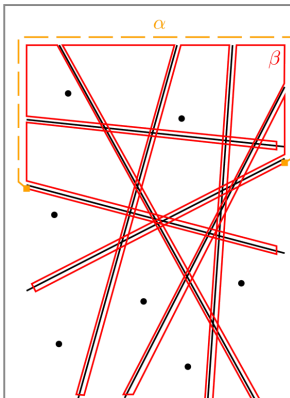


A path in a planar region and a lift to the universal cover.
From Gao et al. [SOCG 1998]

- Shaodi Gao, Mark Jerrum, Michael Kaufmann, Kurt Mehlhorn, Wolfgang Rülling, and Christoph Storb. On continuous homotopic one-layer routing. *Proceedings of the 4th Annual Symposium on Computational Geometry*, 392–402, 1988.
- John Hershberger and Jack Snoeyink. [Computing minimum length paths of a given homotopy class.]() *Computational Geometry: Theory and Applications* 4(2):63–97, 1994. *[The algorithm described in class for computing shortest homotopic paths in triangulated polygons with holes in O(nk) time.]*
- Der-Tsai Lee and Franco P. Preparata. Euclidean shortest paths in the presence of rectilinear barriers. *Networks* 14:393–410, 1984. *[The funnel algorithm again; the paper actually describes an algorithm to compute the tree of shortest paths from any point to all vertices of a simple polygon in O(n log n) time.]*
- Charles E. Leiserson and F. M. Maley. Algorithms for routing and testing routability of planar VLSI layouts. *Proceedings of the 17th Annual ACM Symposium on Theory of Computing*, 69–78, 1985.

Thu, Sep 3
Tue, Sep 8
[[notes]()]

Testing homotopy between paths in the punctured plane: sentinel points, monotone paths, vertical ranking, rectified paths, sliding brackets



Reduction from Hopcroft's problem to contractibility.
From Cabello et al. [DCG 2004]

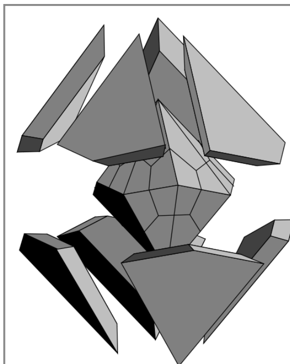
- Sergei Bespamyatnikh. [Computing homotopic shortest paths in the plane.]() *Journal of Algorithms* 49:284–303, 2003.
- Sergei Bespamyatnikh. Encoding homotopy of paths in the plane. *Proceedings of the 6th Latin American Symposium on Theoretical Informatics* , 329–338. *Lecture Notes in Computer Science* 2976, Springer-Verlag, 2004.
- Sergio Cabello, Yuanxin Liu, Andrea Mantler, and Jack Snoeyink. [Testing homotopy for paths in the plane.]() *Discrete & Computational Geometry* 31(1):61–81, 2004. *[The algorithm described in class to check whether two simple paths are homotopic in the punctured plane in O(n log n) time. Distinguishes between tack, pin, and pushpin models; all followup papers use the pin model.]*
- Bernard Chazelle. An algorithm for segment-dragging and its implementation. *Algorithmica* 3:205–221, 1988. *[Describes a data structure for bracket queries. Given a set of n points in the plane, the data structure uses O(n) space, can be built in*

O(n log n) time, and can answer queries of the following form in O(log n) time: Report the leftmost point, if any, in a three-sided rectangle that is open on the right.]

- Alon Efrat, Stephen Kobourov, and Anna Lubiw. Computing homotopic shortest paths efficiently. *Computational Geometry: Theory and Applications* 35(3):162–172, 2006.

Thu, Sep 10 [notes]

Regular homotopy, winding and turning numbers, the Whitney-Graustein theorem, hexahedral meshing, cube complexes for balls



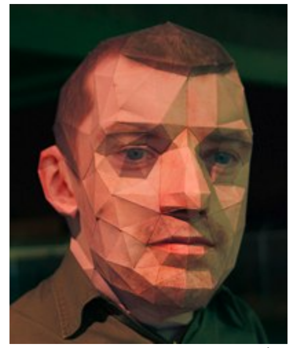
A buffer of hexahedra around a subdivided tetrahedral mesh. From Eppstein [CGTA 1999]

- Marshall Bern, David Eppstein, and Jeff Erickson. Flipping cubical meshes. *Engineering with Computers* 18(3):173-187, 2002.
- David Eppstein. Linear complexity hexahedral mesh generation. *Computational Geometry: Theory & Applications* 12:3–16, 1999.
- Kurt Mehlhorn and Chee-Keng Yap. Constructive Whitney-Graustein theorem: or how to untangle closed planar curves. *SIAM Journal on Computing* 20(4):603–621, 1991. *[Given two polygons with the same turning number, constructs a regular homotopy from one to the otehr consisting of $O(n^2)$ vertex translations. One of the first computational topology papers from the computational geometry community.]*
- Scott A. Mitchell. A characterization of the quadrilateral meshes of a surface which admit a compatible hexahedral mesh of the enclosed volume. *Proceedings of the 13th Annual Symposium on Theoretical Aspects of Computer Science (STACS `96)*, 456–476. *Lecture Notes in Computer Science* 1046, Springer, 1996. *[Proves that any even quad mesh of the sphere can be extended to a hex mesh of the enclosed ball.]*
- Peter Murdoch, Steven Benzley, Ted Blacker, and Scott A. Mitchell. The spatial twist continuum: A connectivity based method for representing all-hexahedral finite element meshes. *Finite Elements in Analysis and Design* 28:137-149, 1997. *[Investigates the arrangement of surfaces dual to any hexahedral mesh.]*
- Tahl Nowik. Complexity of planar and spherical curves. *Duke Journal of Mathematics* 148(1):107–118, 2009. *[Given two regular curves on the plane or the sphere, each with at most n self-intersection points, $\Theta(n^2)$ Whitney moves are always sufficient and sometimes necessary to transform one curve into the other.]*
- William P. Thurston. Hexahedral decomposition of polyhedra. Posting to sci.math, October 25, 1993. *[Proves that any even quad mesh of the sphere can be extended to a hex mesh of the enclosed ball.]*
- Gert Vegter. Kink-free deformation of polygons. *Proceedings of the 5th Annual Symposium on Computational Geometry*, 61–68, 1989. *[Given two polygons with the same turning number, constructs a regular homotopy from one to the otehr consisting of O(n) vertex translations.]*
- Hassler Whitney. On regular closed curves in the plane. *Compositio Mathematica* 4:276–284, 1937.

❖ Curves on Surfaces ❖

Tue, Sep 15
[notes]
HW 1 released

combinatorial 2-manifolds, polygonal schemas, rotation systems, barycentric subdivision, duality, equivalence with topological 2-manifolds

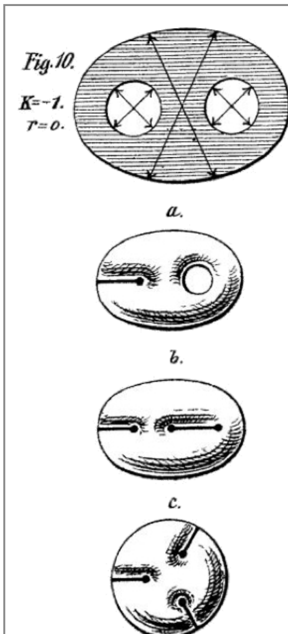


Eric Testroete's papercraft self-portrait (2009)

- Max Dehn and Poul Heegaard. Analysis situs. *Enzyklopädie der mathematischen Wissenschaften mit Einschluß ihrer Anwendungen* III.AB(3):153–220, 1907. *[The first formal development of combinatorial topology.* ***Defines*** *surfaces as the underlying space of a schema of triangles.]*
- Jack Edmonds. A combinatorial representation for polyhedral surfaces. *Notices of the American Mathematical Society* 7:646, 1960. *[Proposes (unsigned) rotation systems for simple graphs on orientable surfaces.]*
- Jonathan L. Gross and Seth R. Alpert. The topological theory of current graphs. *Journal of Combinatorial Theory, Series B* 17:218–233, 1974. *[Generalized rotation systems to multigraphs, by storing permutations of edges instead of permutations of neighbors.]*
- Lothar Heffter. Über das Problem der Nachbargebiete. *Mathematische Annalen* 38(4):477–508, 1891. *[Used polygonal schemata to study colorability of graphs on surfaces.]*
- Béla Kerékjártó. *Vorlesung über Topologie*. Springer-Verlag, 1923. *[Proved that every topological 2-manifold has a triangulation, independently of Radó.]*
- Tibor Radó. Über den Begriff der Riemannschen Fläche. *Acta Litterarum ac Scientiarum, Szeged* 2:101–121, 1925. *[Proved that every topological 2-manifold has a triangulation, independently of Kerékjártó.]*

Thu, Sep 17
[notes]

surface classification, attaching handles and Möbius bands, contracting and deleting edges, tree-cotree decomposition, system of loops, "Oilers' formula"



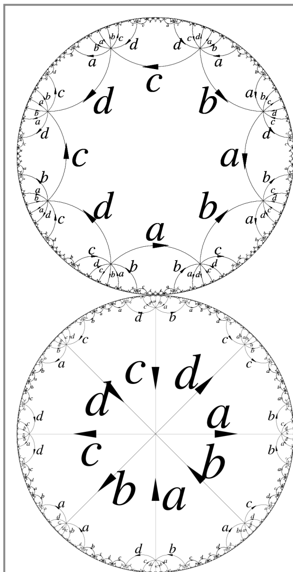
Dyck's surface $\Sigma(1,1)=\Sigma(0,3)$. adapted from Dyck [Math. Ann. 1888]

- James W. Alexander, II. Normal forms for one- and two-sided surfaces. *Annals of Mathematics, Second Series* 16(1/4):158–161, 1914-1915. *[Defined the now-standard canonical polygonal schemata for surfaces used in Brahana's proof, and sketches a proof that any polygonal schema can be transformed into a canonical schema. Alexander calls the canonical schema for orientable surfaces (aba'b'cdc'd'...) "well-known".]*
- Henry R. Brahana. Systems of circuits on two-dimensional manifolds. *Annals of Mathematics* 23(2):144–168, 1922. *[The first universally acknowledged proof of the (combinatorial) surface classification theorem. This is the proof that appears in almost every topology textbook.]*
- Max Dehn and Poul Heegaard. Analysis situs. *Enzyklopädie der mathematischen Wissenschaften mit Einschluß ihrer Anwendungen* III.AB(3):153–220, 1907. *[The first formal development of combinatorial topology. Includes a (sketchy) proof of the (combinatorial) surface classification theorem.]*

- Walther von Dyck. Beiträge zur Analysis situs I. Aufsatz. Ein- und zweidimensionale Mannigfaltigkeiten. *Mathematische Annalen* 32(4):457–512, 1888. *[The first complete statement of the (combinatorial) surface classification theorem. The classification of orientable surfaces was already known (but not proved) by Riemann (1851), Möbius (1863), Jordan (1866), and Klein (1882), among others.]*
- Simon A. J. l'Huilier. Démonstration immédiate d'un théorème fondamental d'Euler sur les polyhèdres, et exception dont ce théorème est susceptible. *Mémoires de l'Académie Impériale des Sciences de Saint-Petersbourg* 4:271–301, 1811. *[The first generalization of Euler's formula to polyhedra with non-zero genus: V-E+F=2-2g.]*
- Carsten Thomassen. The Jordan-Schönflies theorem and the classification of surfaces. *The American Mathematical Monthly* 99(2):116–131, 1992. *[A **completely** self-contained proof of the surface classification theorem, including proofs of the necessary portions of the Jordan-Schönflies and Kerékjártó-Radó theorems.]*
- George Francis and Jeff Weeks. Conway's ZIP proof. *The American Mathematical Monthly* 106(5):393–399, 1999. *[A short proof of the (combinatorial) surface classification theorem. Despite its name, the proof includes a few irrelevancies.]*
- Gert Vegter and Chee-Keng Yap. Computational complexity of combinatorial surfaces. *Proceedings of the 6th Annual Symposium on Computational geometry*, 102–111, 1990. *[Brahana's proof can be transformed into an algorithm to transform any system of loops into a canonical system of loops (as defined by Alexander) in $O(g^2)$ time. Vegter and Yap improve the running time to $O(g \log g)$ using a more careful choice of transformations and data structures.]*

Tue, Sep 22
Thu, Sep 24
[notes]

fundamental groups, group presentations, one-relator presentation from any system of loops, universal covers, hyperbolic tilings, Dehn's contractibility algorithm, compressed crossing sequences



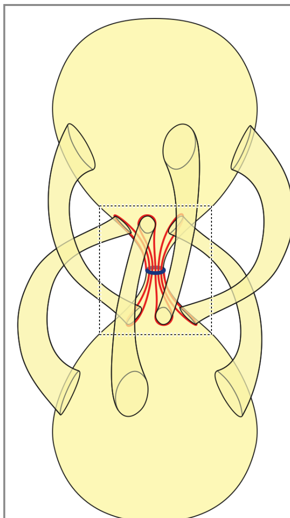
The Cayley graph of the fundamental group of the double torus.
From Yann Oliver

- Max Dehn. Über unendliche diskontinuierliche Gruppen. *Mathematische Annalen* 71(1):16–144, 1911. *[Describes the word problem, the conjugacy problem, and the isomorphism problem for infinite discrete groups.]*
- Max Dehn. Transformation der Kurven auf zweiseitigen Flächen. *Mathematische Annalen* 72(3):413–421, 1912. *[Solves the word problem and the conjugacy problem for fundamental groups of orientable 2-manifolds, using hyperbolic geometry. One of the foundational results of combinatorial group theory.]*
- Tamal K. Dey and Sumanta Guha. Transforming curves on surfaces. *Journal of Computer and System Sciences* 58:297–325, 1999. *[Describes linear-time algorithms to determine whether a cycle on an combinatorial surface is contractible (the word problem in the fundamental group), or whether two cycles are freely homotopic (the conjugacy problem)]*
- Martin D. Greendlinger. Dehn's algorithm for the word problem. *Communications in Pure and Applied Mathematics* 13:67–83, 1960. *[Generalizes Dehn's algorithm for the word problem to abstract groups satisfying a certain small cancellation condition C(1/6).]*

- Martin D. Greendlinger. On Dehn's algorithm for the conjugacy and word problems with applications. *Communications in Pure and Applied Mathematics* 13:641–677, 1960. *[Generalizes Dehn's algorithm for the conjugacy problem to abstract groups satisfying a certain small cancellation condition C'(1/8).]*
- Arye Juhàsz. Solution of the conjugacy problem in one-relator groups. *Algorithms and Classification in Combinatorial Group Theory* (Gilbert Baumslag and Charles F. Miller III, editors), 69–81. *Mathematical Science Research Institute Publications* 23, Springer, 1992.
- Roger D. Lyndon and Paul E. Schupp. *Combinatorial Group Theory*. Springer-Verlag, 2001. *[The standard textbook.]*
- Walter Magnus. Das Identitätsproblem für Gruppen mit einer definierenden Relation. *Mathematische Annalen* 106(1):295–307, 1932. *[Solves the word problem for arbitrary groups with a one-relator presentation.]*

Tue, Sep 29
Thu, Oct 1
[notes]

shortest non-contractible cycles, three-path condition, modified Dijkstra, crossing shortest paths, cutting into disks, cut graphs, crossing sequences



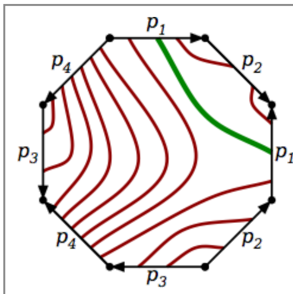
The shortest splitting cycle can cross a shortest path Ω(g) times.
From Chambers et al. [CGTA 2008]

- Sergio Cabello. Many distances in planar graphs. *Proceedings of the 17th Annual ACM-SIAM Symposium on Discrete Algorithms, 1213–1220, 2006.*
- Sergio Cabello and Erin W. Chambers. Multiple source shortest paths in a genus g graph. *Proceedings of the 18th Annual ACM-SIAM Symposium on Discrete Algorithms*, 89–97, 2007.
- Sergio Cabello and Bojan Mohar. Finding shortest non-separating and non-contractible cycles for topologically embedded graphs. *Discrete & Computational Geometry* 37(2):213–235, 2007.
- Jeff Erickson and Sariel Har-Peled. Optimally cutting a surface into a disk. *Discrete & Computational Geometry* 31:37–59, 2004.
- Jeff Erickson and Kim Whittlesey. Greedy optimal homotopy and homology generators. *Proceedings of the 16th Annual ACM-SIAM Symposium on Discrete Algorithms, 1038–1046, 2005.*
- Greg Frederickson. Fast algorithms for shortest paths in planar graphs, with applications. *SIAM Journal on Computing* 16(6):1004–1022, 1987.
- Martin Kutz. Computing shortest non-trivial cycles on orientable surfaces of bounded genus in almost linear time. *Proceedings of the 22nd Annual Symposium on Computational Geometry*, 430–438, 2006.
- Carsten Thomassen. Embeddings of graphs with no short noncontractible cycles. *Journal of Combinatorial Theory, Series B* 48:155–177, 1990.
- John Reif. Minimum *s*-*t* cut of a planar undirected network in $O(n \log^2(n))$ time. *SIAM Journal on Computing* 12(1):71–81, 1983.

Tue, Oct 6
Thu, Oct 8
no notes
(see paper)

minimum cuts in surface graphs: boundary subgraphs, $\mathbb{Z}_2$-homology, shortest-path crossing bounds, crossing (parity) vectors, weighted triangulations

- Erin W. Chambers, Jeff Erickson, and Amir Nayyeri. Minimum cuts and shortest homologous cycles. *Proceedings of the 25th Annual Symposium on Computational Geometry*, 377–385, 2009.
- Erin W. Chambers, Éric Colin de Verdière, Jeff Erickson, Francis Lazarus, and Kim Whittlesey. Splitting (complicated) surfaces is hard. *Computational Geometry: Theory & Applications* 41(1-2):94–110, 2008.

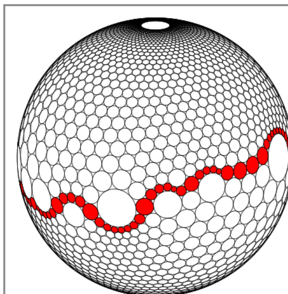


Two cycles on a double-torus.
From Chambers et al. [SoCG 2009]

❖ Topological graph theory ❖

Tue, Oct 13
Thu, Oct 15
[notes]
HW2 released

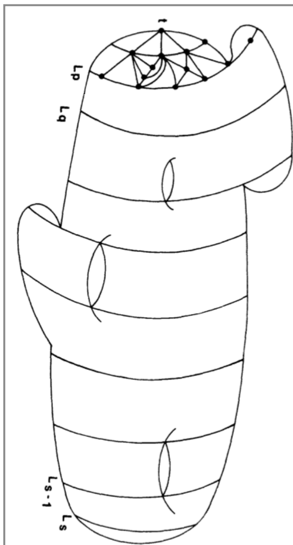
graph separators: Menger's theorem, planar graphs, disk packing, center points, greedy tree-cotree decomposition, level slicing



A small balanced separator of a planar graph.
From Ken Stephenson

- Michael O. Albertson and Joan P. Hutchinson. On the independence ratio of a graph. *Journal of Graph Theory* 2(1):1–8, 1978.
- Lyudmil Aleksandrov and Hristo Djidjev. Linear algorithms for partitioning embedded graphs of bounded genus. *SIAM Journal on Discrete Mathematics* 9(1):129–150, 1996.
- Noga Alon, Paul Seymour, and Robin Thomas. Planar separators. *SIAM Journal on Discrete Mathematics* 7(4):184–193, 1994.
- E. M. Andreev. Convex polyhedra in Lobačevskiĭ space. *Math. USSR Sbornik 10:413–440, 1970.*
- E. M. Andreev. On convex polyhedra of finite volume in Lobačevskiĭspace. *Math. USSR Sbornik* 12(2):270–259, 1970.
- Hristo N. Djidjev. A separator theorem. *Comptes Rendus de l' Académie Bulgare des Sciences* 34:643–645, 1981.
- David Eppstein. Dynamic generators of topologically embedded graphs. *Proceedings of the 14th Annual ACM-SIAM Symposium on Discrete Algorithms*, 599–608, 2003.
- John R. Gilbert, Joan P. Hutchinson, and Robert E. Tarjan. A separator theorem for graphs of bounded genus. *Journal of Algorithms* 5(3):391–407, 1984.
- Paul Koebe. Kontaktprobleme der Konformen Abbildung. *Berichte über die Verhandlungen der Sächsischen Akademie der Wissenschaften, Leipzig, Math.-Phys. Klasse* 88:141–164, 1936.
- Richard J. Lipton and Robert E. Tarjan. A separator theorem for planar graphs. *SIAM Journal on Applied Mathematics* 36(2):177–189, 1979.

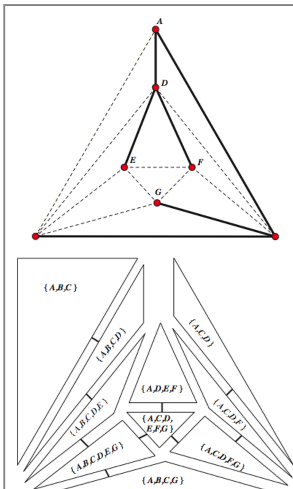
- Kurt Menger. Zur allgemeinen Kurventheorie. *Fundamenta Mathematica* 10:96–115, 1927.
- Warren D. Smith and Nicolas C. Wormald. [Geometric separator theorems and applications.](#) *Proceedings of the 39th Annual IEEE Symposium on Foundations of Computer Science*, 232–243, 1998.
- Dan Spielman and Shang-Hua Teng. Disk packings and planar separators. *Proceedings of the 12th Annual Symposium on Computational Geometry*, 349–358, 1996.



Cutting a surface into slices.
From Hutchinson [SIDM 1998]

Tue, Oct 20 **[[notes](#)]**

tree decompositions, treewidth, Baker's slicing technique, treewidth vs. diameter, PTAS for maximum independent set



A tree decomposition for a planar triangulation.
From Eppstein [JGAA 1999]

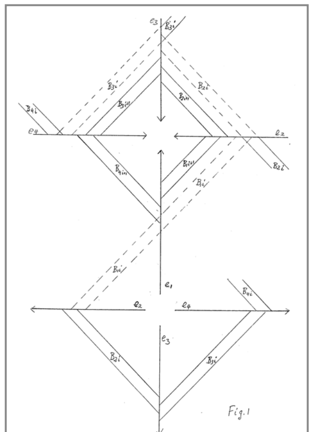
- Stefan Arnborg and Andrzej Proskurowski. Linear time algorithms for {NP}-hard problems restricted to partial *k*-trees. *Discrete Applied Mathematics* 23:11–24, 1989.
- Brenda S. Baker. Approximation algorithms for NP-complete problems on planar graphs. *Journal of the Association for Computing Machinery* 41(1):153–180, 1994.
- Umberto Bertelè and Francesco Brioschi. *Nonserial Dynamic Programming. Academic Press, New York, 1972.*
- Hans L. Bodlaender. A linear-time algorithm for finding tree-decomposition of small treewidth. *SIAM Journal on Computing 25:1305--1317, 1996.*
- David Eppstein. Subgraph isomorphism in planar graphs and related problems. *Journal of Graph Algorithms and Applications* 3(3):1—27, 1999.
- David Eppstein. Diameter and treewidth in minor-closed families. *Algorithmica* 27:275—291, 2000.
- Neil Robertson and Paul D Seymour. Graph minors. II. Algorithmic aspects of tree-width. *Journal of Algorithms 7(3):309–322, 1986.*

Thu, Oct 22 **[[notes](#)]**

Graph minors: [definitions](#), Kuratowski-Wagner, examples, [the Graph Minor Theorem](#), minor-closed families, finite obstruction sets, membership algorithm, combinatorial properties, planar minor or bounded treewidth, decomposition theorem, lots of references but no proofs

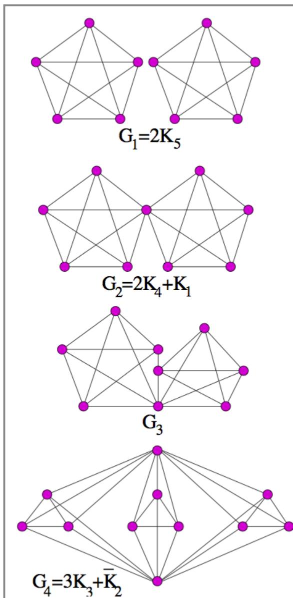
- Noga Alon, Paul D. Seymour, and Robin Thomas. A separator theorem for nonplanar graphs. *Journal of the American Mathematical Society* 3(4):801–808, 1990.

- Erik D. Demaine and MohammadTaghi Hajiaghayi. Linearity of grid minors in treewidth with applications through bidimensionality. *Combinatorica* 28(1):19--36, 2008.



Interlaced parallel bundles. From Vollmerhaus 1987

- Erik D. Demaine, MohammadTaghi Hajiaghayi, and Ken-ichi Kawarabayashi. Algorithmic graph minor theory: Decomposition, approximation, and coloring. *Proceedings of the 46th IEEE Symposium on Foundations of Computer Science*, 637–646, 2005.
- Matt DeVos, Guoli Ding, Bogdan Oporowski, Daniel P. Sanders, Bruce Reed, Paul Seymour, and Dirk Vertigan. Excluding any graph as a minor allows a low tree-width 2-coloring. *Journal of Combinatorial Theory, Series B* 91(1):25--41, 2004.
- Alexandr V. Kostochka. Lower bound of the Hadwiger number of graphs by their average degree. *Combinatorica* 4(4):307--316, 1984.



The obstruction set for torus graphs with no $K_{3,3}$ minor. From Gagarin et al. 2005

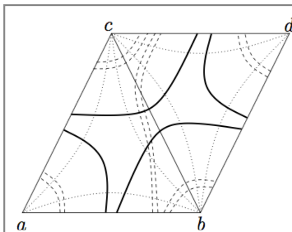
- Casimir Kuratowski. Sur the problème des courbes gauches en Topologie. *Fundamenta Mathematicae* 15:271–283, 1930.
- Serge Plotkin, Satish Rao, and Warren D. Smith. Shallow excluded minors and improved graph decompositions. *Proceedings of the 5th Annual ACM-SIAM Symposium on Discrete Algorithms, 462–470, 1994.*
- Neil Robertson and Paul D. Seymour. Generalizing Kuratowski's theorem. *Congressus Numerantum* 45:129–138, 1984.
- Neil Robertson and Paul D. Seymour. Graph minors. V. Excluding a planar graph. *Journal of Combinatorial Theory, Series B* 41(1):92–114, 1986.
- Neil Robertson and Paul D. Seymour. Graph minors. XIII. The disjoint paths problem. *Journal of Combinatorial Theory, Series B* 63:65–110, 1995.
- Neil Robertson and Paul D. Seymour. Graph minors. XVI. Excluding a non-planar graph. *Journal of Combinatorial Theory, Series B* 89:43–76, 2003.
- Neil Robertson and Paul D. Seymour. Graph minors. XX. Wagner's conjecture. *Journal of Combinatorial Theory, Series B* 92:325--357, 2004.
- Neil Robertson, Paul D. Seymour, and Robin Thomas. Quickly excluding a planar graph. *Journal of Combinatorial Theory, Series B* 62(2):232–348, 1994.
- Neil Robertson, Paul D. Seymour, and Robin Thomas. Sachs' linkless embedding conjecture. *Journal of Combinatorial Theory, Series B* 64(2):185–227, 1995.
- Andrew Thomason. The extremal function for complete minors. *Journal of Combinatorial Theory, Series B* 81(2):318--338, 2001.

- Kurt Wagner. Über eine Erweiterung des Satzes von Kuratowski. *Deutsche Mathematik* 2:280–285, 1937.

❖ Normal curves and surfaces ❖

Tue, Oct 27
Thu, Oct 29
[notes]

Normal curves, corner and edge coordinates, exponential compression, straight-line programs, word equations, testing connectivity, counting components, testing isotopy

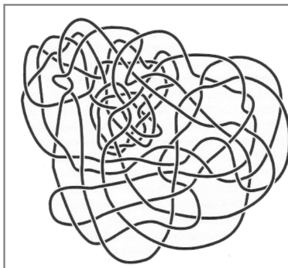


Representatives of two isotopy classes of curves. From Schaefer et al. [CCCG 2008]

- Ian Agol, Joel Hass, and William P. Thurston. The computational complexity of knot genus and spanning area. *Transactions of the American Mathematical Society* 358(9):3821–3850, 2006.
- Wolfgang Haken. Theorie der Normalflächen: Ein Isotopiekriterium für den Kreisknoten. *Acta Mathematica* 105:245–375, 1961.
- Helmuth Kneser. Geschlossene Flächen in dreidimensionalen Mannigfaltigkeiten. *Jahresbericht der Deutscher Mathematiker-Vereinigung* 38:248–260, 1929.
- Markus Lohrey. Word problems and membership problems on compressed words. *SIAM Journal on Computing 35(5):1210–1240, 2006.*
- Masamichi Miyazaki, Ayumi Shinohara, Masayuki Takeda. An improved pattern matching algorithm for strings in terms of straight-line programs. *Journal of Discrete Algorithms* 1(1):187–204, 2000.
- Wojciech Plandowski and Wojciech Rytter. Application of Lempel-Ziv encodings to the solution of word equations. *Proceedings of the 25th International Conference on Automata, Languages, and Programming*, 731–742, 1998. *Lecture Notes in Computer Science* 1443, Springer-Verlag.
- Marcus Schaefer, Eric Sedgwick, and Daniel Štefankovič. Algorithms for normal curves and surfaces. *Proceedings of the 8th International Conference on Computing and Combinatorics*, 370–380. *Lecture Notes in Computer Science* 2387, Springer, 2002.
- Marcus Schaefer, Eric Sedgwick, and Daniel Štefankovič. Computing Dehn twists and geometric intersection numbers in polynomial time. *Proceedings of the 20th Canadian Conference on Computational Geometry*, 111–114, 2008. Full version available as Technical Report 05-009, Computer Science Department, DePaul University, April 2005.
- Saul Schleimer. Polynomial-time word problems. *Commentarii Mathematici Helvetici* 83(4):741–765, 2008.

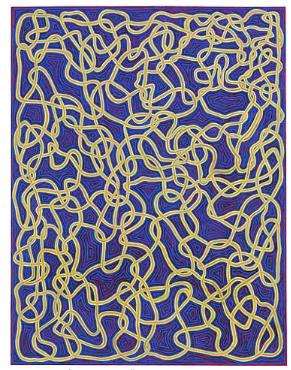
Tue, Nov 4
[notes]

Normal surfaces, knots, ambient isotopy, spanning disk, triangulating knot manifolds, Haken normal cone, fundamental surfaces, vertex surfaces, unknot ∈ NP


Wolfgang Haken's "Gordian unknot" From Ian Agol

- Ian Agol, Joel Hass, and William P. Thurston. The computational complexity of knot genus and spanning area. *Transactions of the American Mathematical Society* 358(9):3821–3850, 2006.

- Max Dehn. Über die Topologie des dreidimensionalen Raumes. *Mathematische Annalen*. 69(1):137–168, 1910.
- Wolfgang Haken. Theorie der Normalflächen: Ein Isotopiekriterium für den Kreisknoten. *Acta Mathematica* 105:245–375, 1961.
- Masao Hara, Seiichi Tani, and Makoto Yamamoto. UNKNOTTING is in AM ∩ co-AM. *Proceedings of the 16th Annual ACM-SIAM Symposium on Discrete Algorithms*, 359–364, 2005.
- Joel Hass, Jeffrey C. Lagarias, and Nicholas Pippenger. The computational complexity of knot and link problems. *Journal of the ACM 46(2):185–211, 1999.*

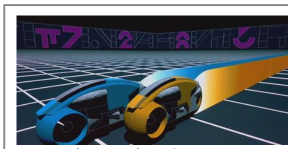


"Unknot" by James Siena (2005)

- William Jaco and Jeffrey L. Tollefson. Algorithms for the complete decomposition of a closed 3-manifold. *Illinois Journal of Mathematics* 39(3):358–406, 1995.
- Helmuth Kneser. Geschlossene Flächen in dreidimensionalen Mannigfaltigkeiten. *Jahresbericht der Deutscher Mathematiker-Vereinigung* 38:248–260, 1929.
- Christos D. Papakyriakopoulos. On Dehn's lemma and the asphericity of knots. *Annals of Mathematics* 66:1–26, 1957.
- Michael O. Rabin. Recursive unsolvability of group-theoretic problems. *Annals of Mathematics* 67:172–194, 1958.
- Marcus Schaefer, Eric Sedgwick, and Daniel Štefankovič. Algorithms for normal curves and surfaces. *Proceedings of the 8th International Conference on Computing and Combinatorics*, 370–380. *Lecture Notes in Computer Science* 2387, Springer, 2002.

❖ Project proposals ❖

Thu, Nov 6 | **Problem summary presentations** — no regular lecture

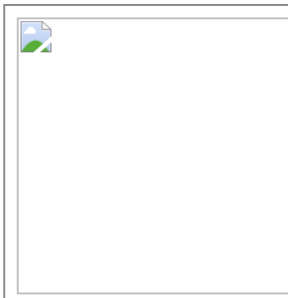


Light cycles from *Tron* (1982)

❖ Complexes and Homology ❖

Tue, Nov 9 [notes] | Cell complex definitions: abstract simplicial complexes, geometric simplicial complexes, polytopal complexes, cubical complexes, Δ-complexes, Closure-finite Weak-topology (CW) complexes

- See notes for references (for now).



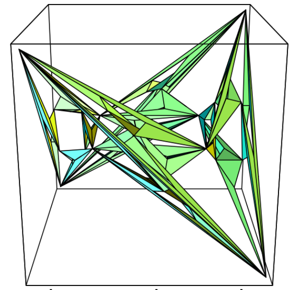
From Jonathan Puckey's

"Delaunay Raster" series
(2009)

Thu, Nov 11
[notes]

Cell complex examples: Alexandroff-Čech complexes, Vietoris-Rips complexes, Delaunay complexes, alpha complexes; state/configuration complexes; monotone graph property complexes; presentation complexes and undecidability

- See notes for references (for now).

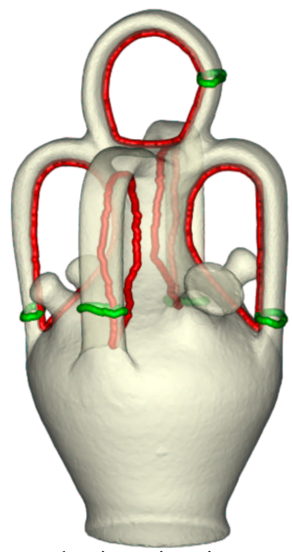


A three-round test-and-set protocol complex from Herlihy and Rajsbaum (1995)

Tue, Nov 16
Thu, Nov 18
[notes]

Simplicial homology: chains, boundary homomorphisms, cycles, boundaries, homology classes, homology groups, Euler-Poincaré formula, invariance and the Hauptvermutung, homology of 2-manifolds from polygonal schemata, the Smith-Poincaré reduction algorithm

- See notes for references (for now).



A first homology basis from Dey et al. [SIGGRAPH 2008]

Tue, Nov 23
Thu, Nov 25

Thanksgiving break
no classes!

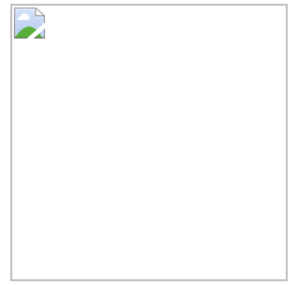
Tue, Dec 1
~~[no notes yet]~~

Persistent homology: motivation (robust topological inference), images of maps between homology groups induced by inclusion, filtrations, positive and negative simplices, incremental computation, simplex pairing, persistence barcodes/diagrams, extracting persistent homology groups, time-based persistence

A topological bar code. Get it?
From Rob Ghrist

Thu, Dec 3
[no notes; see [paper]]

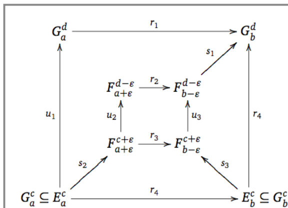
[The paper Amir and I submitted to SOCG yesterday], which has nothing to do with homology but I'm *very* tired and the voices in my head won't let me think about anything abelian woah is that a wombat?



Five streets.

Tue, Dec 8
~~[no notes yet]~~

Persistent homology continued: persistence modules, graded $\mathbb{Z}_2[t]$-module decompositions (via Smith-Poincaré reduction), barcodes/diagrams revisited, tame functions, sublevel sets, barcodes/diagrams rerevisited, Hausdorff and bottleneck distance between point multisets, hey look an actual commutative diagram, quadrants and boxes, stability of persistence diagrams

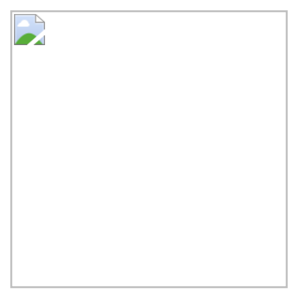


A commutative diagram from Cohen-Steiner et al. [*DCG* 2007]

❖ Final Project Presentations ❖

Wed Dec 16 and Thu Dec 17

[Final project presentations]



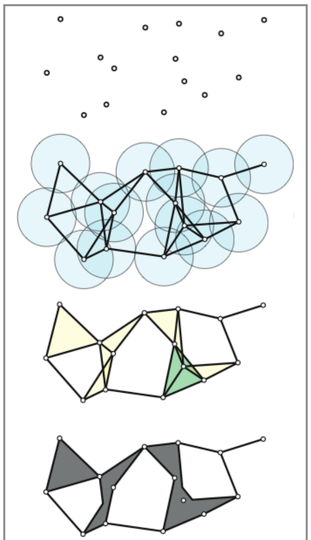
Nested Klein bottles, by Alan Bennett (1995)

❖ Related Topics ❖

These are topics that are definitely within the scope of the class, but that I ~~do not plan to~~ did not cover this semester. This is an *incredibly* incomplete list; please don't be insulted if your favorite result is missing! I expect to cover a *very* different set of material the next time I teach this class.

☆ **Vietoris-Rips complexes and sensor coverage**

- Erin W. Chambers, Vin de Silva, Jeff Erickson, and Robert Ghrist. [Vietoris-Rips complexes of planar point sets]. *Discrete & Computational Geometry, in press, 2009.*
- Erin W. Chambers, Jeff Erickson, and Pratik Worah. [Testing contractibility in planar Rips complexes]. *Proceedings of the 24th Annual ACM Symposium on Computational Geometry*, 251–259, 2008.



A point set, its unit-distance graph, its Vietoris-Rips complex, and its shadow. From Chambers et al. [SOCG 2008]

- Vin de Silva and Robert Ghrist. Coordinate-free coverage in sensor networks with controlled boundaries via homology. International Journal of Robotics Research 25(12):1205–1222, 2006.
- Vin de Silva and Robert Ghrist. Coverage in sensor networks via persistent homology. *Algebraic & Geometric Topology* 7:339–358, 2007.
- Alireza Tahbaz-Salehi and Ali Jadbabaie. Distributed computation of a sparse cover in sensor networks without location information. *Proceedings of the 42nd Annual Conference on Information Sciences and Systems*, 1285d–-1290, 2008.
- Alireza Tahbaz-Salehi and Ali Jadbabaie. Distributed coverage verification in sensor networks without location information. *Proceedings of the 47th IEEE Conference on Decision and Control*, 4170–4176, 2008.

☆ **Surface encoding and compression: Edgebreaker, Schneyder woods**

-
- Luca Castelli Aleardi, Éric Fusy, and Thomas Lewiner. Schnyder woods for higher genus triangulated surfaces, with applications to encoding. *Discrete & Computational Geometry* 42(3):489–516, 2009.
- Helio Lopes, Jarek Rossignac, Alla Safonova, Andrzej Szymczak and Geovan Tavares. Edgebreaker: A simple compression algorithm for surfaces with handles. *Computers & Graphics International Journal* 27(4):553–567, 2003.
- Jarek Rossignac. Edgebreaker: Connectivity compression for triangle meshes. *IEEE Transactions on Visualization and Computer Graphics* 5(1):47–61, 1999.

☆ Knot and link invariants: crossing number, bridge number, twist, writhe, genus, Alexander-Conway, Jones, Homflypt/Flypmoth/Thomflyp/skein
☆ Discrete Morse theory, Reeb graphs, Morse-Smale complexes, harmonic quadrangulation
☆ Friedman's homology through combinatorial Laplacians
☆ Zig-zag homology
☆ Maximum flows on surface-embedded graphs