(Untitled, Till Rickert, Shift 2005 Calendar.)

CS 274 Computational Geometry

Jonathan Shewchuk

Spring 2005
Mondays and Wednesdays, 1:00-2:30 pm
320 Soda Hall

Combinatorial geometry: Polygons, polytopes, triangulations, planar and spatial subdivisions. Constructions: triangulations of polygons, convex hulls, intersections of halfspaces, Voronoi diagrams, Delaunay triangulations, arrangements of lines and hyperplanes, Minkowski sums; relationships among them. Geometric duality and polarity. Numerical predicates and constructors. Upper Bound Theorem, Zone Theorem.

Algorithms and analyses: Sweep algorithms, incremental construction, divide-and-conquer algorithms, randomized algorithms, backward analysis, geometric robustness. Construction of triangulations, convex hulls, halfspace intersections, Voronoi diagrams, Delaunay triangulations, arrangements, Minkowski sums.

Geometric data structures: Doubly-connected edge lists, quad-edges, face lattices, trapezoidal maps, history DAGs, spatial search trees (a.k.a. range search), binary space partitions, visibility graphs.

Applications: Line segment intersection and overlay of subdivisions for cartography and solid modeling. Triangulation for graphics, interpolation, and terrain modeling. Nearest neighbor search, small-dimensional linear programming, database queries, point location queries, windowing queries, discrepancy and sampling in ray tracing, robot motion planning.

Here are Homework 1, Homework 2, Homework 3, Homework 4, and Homework 5.

The best related sites:

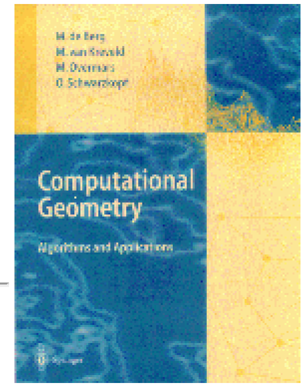
- David Eppstein's **Geometry in Action** and **Geometry Junkyard**.
- Jeff Erickson's **Computational Geometry Pages**.
- Lists of open problems in computational geometry from Erik Demaine et al., Jeff Erickson, and David Eppstein.

Resources for dealing with robustness problems (in increasing order of difficulty):

- My robust predicates page (floating-point inputs, C).
- Chee Yap's **CORE Library** (C/C++).

- David Bailey's extensive **MPFUN** arbitrary precision arithmetic package (floating-point, C++ or Fortran).
- Olivier Devillers' predicates (integer inputs).
- Stefan Näher et al.'s **LEDA** contains several arbitrary precision numerical types, including integers and floating-point (C++). Commercial; you have to pay for it.

Textbook



Mark de Berg, Marc van Kreveld, Mark Overmars, and Otfried Schwarzkopf (presently known as Otfried Cheong), Computational Geometry: Algorithms and Applications, second revised edition, Springer-Verlag, 2000. ISBN # 3-540-65620-0.
Known throughout the community as the Dutch Book.

Lectures

Homeworks will be irregularly assigned, and are due at the start of class on a Wednesday. Homeworks are to be done alone, without help from or discussion with other humans.

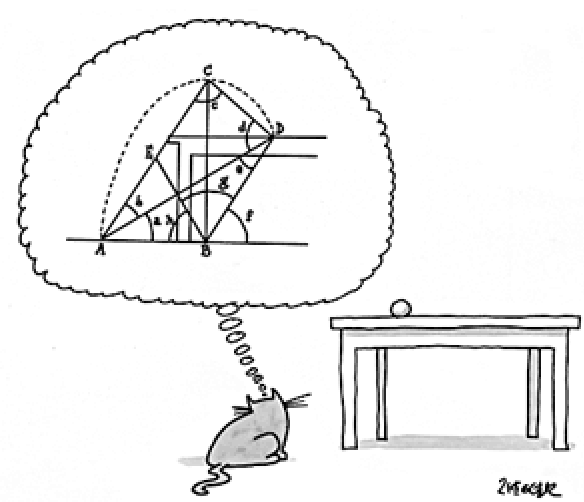
	Topic	**Readings**	**Due Wednesday**
1: January 19	Two-dimensional convex hulls	Chapter 1, Erickson notes	.
2: January 24	Line segment intersection	Sections 2, 2.1	.
3: January 26	Overlay of planar subdivisions	Sections 2.2, 2.3, 2.5	.
4: January 31	Polygon triangulation	Sections 3.2-3.4	.
5: February 2	Delaunay triangulations	Sections 9-9.2	.
6: February 7	Delaunay triangulations	Sections 9.3, 9.4, 9.6	.
7: February 9	Voronoi diagrams	Sections 7, 7.1, 7.3	.
8: February 14	Planar point location	Chapter 6	.
9: February 16	Duality; line arrangements	Sections 8.2, 8.3	Homework 1
February 21	**Presidents' Day**	.	.
10: February 23	Zone theorem; discrepancy	Sections 8.1, 8.4	.
11: February 28	Polytopes	Matoušek Chapter 5	.
12: March 2	Polytopes and triangulations	Seidel Upper Bound Theorem	Homework 2
13: March 7	Small-dimensional linear programming	Sections 4.3, 4.6; Seidel T.R.	.
14: March 9	Small-dimensional linear programming	Section 4.4; Seidel appendix	.
15: March 14, **12:40**	Carlo Séquin on splines, **203 McLaughlin**	Carlo's lecture notes 1	.
16: March 16, **12:40**	Carlo Séquin on subdivision, **203 McLaughlin**	Carlo's lecture notes 2	.
March 21-25	**Spring Recess**		

17: March 28	Higher-dimensional convex hulls	Seidel T.R.; Secs. 11.2 and 11.3	.
18: March 30	Higher-dimensional Voronoi; point in polygon	Secs. 11.4, 11.5; Smid Sec. 2.4.3	Homework 3
19: April 4	k-d trees	Sections 5-5.2	.
20: April 6	Range trees	Sections 5.3-5.6	.
21: April 11	Interval trees	Sections 10-10.1	.
22: April 13	Segment trees	Section 10.3	.
23: April 18	Binary space partitions	Sections 12-12.3	.
24: April 20	Binary space partitions	Sections 12.4, 2.4, BSP FAQ	.
25: April 25	Robot motion planning	Sections 13-13.2	.
26: April 27	Minkowski sums	Sections 13.3-13.5	.
27: May 2	Visibility graphs	Chapter 15; Khuller notes	Homework 4
28: May 4	Geometric robustness	Lecture notes	Project
29: May 9	Constrained triangulations	.	.
May 13	.	.	Homework 5

For January 19, here are Jeff Erickson's lecture notes on two-dimensional convex hulls.

For February 28 and March 2, if you want to supplement my lectures, most of the material comes from Chapter 5 of Jiří Matoušek, Lectures on Discrete Geometry, Springer (New York), 2002, ISBN # 0387953744. He has several chapters online; unfortunately Chapter 5 isn't one of them.

For March 2, I will hand out Raimund Seidel, The Upper Bound Theorem for Polytopes: An Easy Proof of Its Asymptotic Version, Computational Geometry: Theory and Applications **5**:115-116, 1985. Don't skip the abstract: the main theorem and its proof are both given in their entirety in the abstract, and are not reprised in the body at all.



Seidel's linear programming algorithm (March 7 & 9), the Clarkson-Shor convex hull construction algorithm (March 28), and Chew's linear-time algorithm for Delaunay triangulation of convex polygons are reported in Raimund Seidel, Backwards Analysis of Randomized Geometric Algorithms, Technical Report TR-92-014, International Computer Science Institute, University of California at Berkeley, February 1992. Warning: online paper is missing the figures. I'll hand out a version with figures in class.

For March 9, I will hand out the appendix from Raimund Seidel, Small-Dimensional Linear Programming and Convex Hulls Made Easy, Discrete & Computational Geometry **6**(5):423-434, 1991. For anyone who wants to implement the linear programming algorithm, I think this appendix is a better guide than the Dutch Book.

On March 30, I will teach a randomized closest pair algorithm from Section 2.4.3 of Michiel Smid, Closest-Point Problems in Computational Geometry, Chapter 20, Handbook on Computational Geometry, J. R. Sack and J.

Urrutia (editors), Elsevier, pp. 877-935, 2000. Note that this is a long paper, and you only need pages 12-13.

For April 18, here is the BSP FAQ.

For April 27, here are Samir Khuller's notes on visibility graphs.

For May 2, here are my Lecture Notes on Geometric Robustness.

For the Project, read Leonidas J. Guibas and Jorge Stolfi, Primitives for the Manipulation of General Subdivisions and the Computation of Voronoi Diagrams, ACM Transactions on Graphics **4**(2):74-123, April 1985. Feel free to skip Section 3, but read the rest of the paper. See also this list of errors in the Guibas and Stolfi paper, and Paul Heckbert, Very Brief Note on Point Location in Triangulations, December 1994. (The problem Paul points out can't happen in a Delaunay triangulation, but it's a warning in case you're ever tempted to use the Guibas and Stolfi walking-search subroutine in a non-Delaunay triangulation.)

Geometry Applets

These applets can be quite helpful in establishing your geometric intuition for several basic geometric structures and concepts.

- Convex hulls
- Delaunay triangulations
- Voronoi diagrams and Delaunay triangulations I
- Voronoi diagrams and Delaunay triangulations II
- Line sweep
- Fortune's sweep-line Delaunay triangulation algorithm
- Quadtrees of points in the plane

Prerequisites

- CS 170 (Advanced Algorithms) or the equivalent. In particular, you should know and understand amortized analysis; how to solve recurrences; sorting algorithms; graph algorithms like Dijkstra's shortest path algorithm, connected components, and topological sorting; and basic data structures like binary heaps, hash tables, and balanced binary search trees (splay trees or AVL trees or red-black trees or 2-3-4 trees or B-trees). Every one of these will make an appearance at least once.
- A basic course in probability.
- Experience doing mathematical proofs. If you've never taken a class where you did lots of proofs, consider working your way through Bruce Ikenaga's notes and Larry Cusick's notes and exercises.

Grading

- **80%** for the homeworks.
- **20%** for the project: a Delaunay triangulation implementation, or an alternative by arrangement with the instructor.

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(Radiolarian Color Painting. **Ernst Haeckel, zoologist, 1834-1919.**)

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