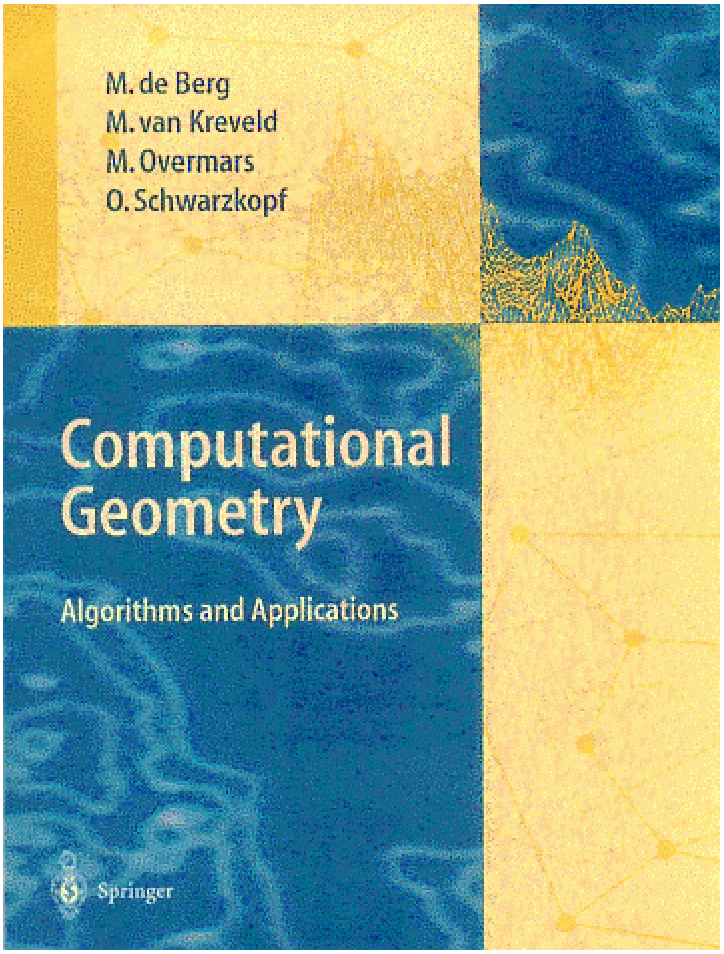

CS 274
Computational Geometry

Jonathan Shewchuk

Spring 2003
Tuesdays and Thursdays, 3:30-5:00 pm
Beginning January 21
405 Soda Hall

Synopsis: Constructive problems in computational geometry: convex hulls, triangulations, Voronoi diagrams, Delaunay triangulations, arrangements of lines and hyperplanes, subdivisions. Relationships among these problems.

Techniques in computational geometry: data structures, incremental construction, divide-and-conquer algorithms, randomized algorithms, backward analysis, geometric robustness. Line segment intersection, planar subdivisions, spatial search trees, visibility graphs, small-dimensional linear programming.

Applications: Nearest neighbor search; triangulation for graphics, interpolation, and terrain modeling; database queries, point locations queries, and windowing queries; collision detection; discrepancy and sampling in ray tracing; robot motion planning; cartography; art gallery theorems.

Here is Homework 1, Homework 2, Homework 3, Homework 4, and Homework 5.

The best related sites:

- David Eppstein's **Geometry in Action** and **Geometry Junkyard**.
- Jeff Erickson's **Computational Geometry Pages**.
- Lists of open problems in computational geometry from Jeff Erickson, David Eppstein, and Erik Demaine et al.

Resources for dealing with robustness problems (in increasing order of difficulty):

- My robust predicates page (floating-point inputs, C).
- Chee Yap's **CORE Library** (C/C++).
- David Bailey's extensive **MPFUN** arbitrary precision arithmetic package (floating-point, Fortran). Look for **mpfun.tex** and **mpfun.f**.
- Olivier Devillers' predicates (integer inputs).
- Stefan Näher et al.'s **LEDA** contains several arbitrary precision numerical types, including integers and floating-point (C++).

Textbook

Mark de Berg, Marc van Kreveld, Mark Overmars, and Otfried Schwarzkopf (presently known as Otfried Cheong), Computational Geometry: Algorithms and Applications, second revised edition, Springer-Verlag, 2000. ISBN # 3-540-65620-0.
Known throughout the community as the Dutch Book.

Lectures

Homeworks will be irregularly assigned, and are due at the start of class on a Thursday. Homeworks are to be done alone, without help from or discussion with other humans or comparable intelligences.

	Topic	**Readings**	**Due Thursday**
1: January 21	Two-dimensional convex hulls	Chapter 1	.
2: January 23	Line segment intersection	Sections 2, 2.1	.
3: January 28	Overlay of planar subdivisions	Sections 2.2, 2.3, 2.5	.
4: January 30	Polygon triangulation	Sections 3.2-3.4	.
5: February 4	Delaunay triangulations	Sections 9-9.2	.
6: February 6	Delaunay triangulations	Sections 9.3, 9.4, 9.6	Homework 1
7: February 11	Marshall Bern on interpolation	.	.
8: February 13	Vladlen Koltun on Davenport-Schinzel sequences	.	.
9: February 18	Delaunay triangulations, Voronoi diagrams	Sections 7, 7.1, 7.3	.
10: February 20	Planar point location	Chapter 6	Homework 2
11: February 25	Small-dimensional linear programming	Sections 4.3, 4.6; Seidel T.R.	.
12: February 27	Leonidas Guibas on kinetic data structures	.	.
13: March 4	Small-dimensional linear programming	Section 4.4; Seidel appendix	.
14: March 6	Duality; line arrangements	Sections 8.2, 8.3	.
15: March 11	Zone theorem; discrepancy	Sections 8.1, 8.4	.
16: March 13	Polytopes and triangulations	.	Homework 3
17: March 18	Carlo Séquin on splines	.	.
18: March 20	Carlo Séquin on splines	.	.
March 24-28	**Spring Recess**	.	.
19: April 1	Polytopes and triangulations	Seidel article; Secs. 11.4 and 11.5	.
20: April 3	Higher-dimensional convex hulls	Seidel T.R.; Secs. 11.2 and	.

		11.3	
21: April 8	k-d trees	Sections 5-5.2	.
22: April 10	Range trees	Sections 5.3-5.6	.
23: April 15	Interval trees	Sections 10-10.1	.
24: April 17	Segment trees	Section 10.3	.
25: April 22	Binary space partitions	Sections 12-12.3	.
26: April 24	Binary space partitions	Sections 12.4, 2.4, BSP FAQ	Homework 4
27: April 29	Vladlen Koltun on Minkowski sums	Sections 13-13.2	.
28: May 1	Robot motion planning	Sections 13.3-13.5	.
May 2	.	.	Project, 5 pm
29: May 6	Visibility graphs	Chapter 15; Khuller notes	.
30: May 8	Geometric robustness	Lecture notes	.
31: May 13	Loose ends	.	.
May 22	.	.	Homework 5, 5 pm

For January 21, here are Jeff Erickson's lecture notes on two-dimensional convex hulls.

Chew's linear-time algorithm for Delaunay triangulation of convex polygons (February 18), Seidel's linear programming algorithm (February 25 & March 4), and the Clarkson-Shor convex hull construction algorithm (April 3) are reported in Raimund Seidel, Backwards Analysis of Randomized Geometric Algorithms, Technical Report TR-92-014, International Computer Science Institute, University of California at Berkeley, February 1992. Warning: online paper is missing the figures. I'll hand out a version with figures in class.

For March 4, I will hand out the appendix from Raimund Seidel's Small-Dimensional Linear Programming and Convex Hulls Made Easy, Discrete & Computational Geometry **6**(5):423-434, 1991. For anyone who wants to implement the linear programming algorithm, this appendix is a better guide than the Dutch Book. Unfortunately, there is no online version.

For April 1, I will hand out Raimund Seidel's The Upper Bound Theorem for Polytopes: An Easy Proof of Its Asymptotic Version, Computational Geometry: Theory and Applications **5**:115-116, 1985. Don't skip the abstract: the main theorem and its proof are both given in their entirety in the abstract, and are not reprised in the body at all. Unfortunately, there is no online version.

For April 24, here is the BSP FAQ.

For May 6, here are Samir Khuller's notes on visibility graphs.

For May 8, here are my Lecture Notes on Geometric Robustness.

For the Project, read Leonidas J. Guibas and Jorge Stolfi, Primitives for the Manipulation of General Subdivisions and the Computation of Voronoi Diagrams, ACM Transactions on Graphics **4**(2):74-123, April 1985. Unfortunately, there is no online version, but I'll hand it out in class. Feel free to skip Section 3, but read the rest of the paper. See also this list of errors in the Guibas and Stolfi paper, and Paul Heckbert, Very Brief Note on Point Location in Triangulations, December 1994. (The problem Paul points out can't happen in a Delaunay triangulation, but it's a warning in case you're ever tempted to use the Guibas and Stolfi walking-search subroutine in a non-Delaunay triangulation.)

Geometry Applets

These applets can be quite helpful in establishing your geometric intuition for several basic geometric structures and concepts.

- Convex hulls
- Delaunay triangulations
- Voronoi diagrams and Delaunay triangulations I
- Voronoi diagrams and Delaunay triangulations II
- The duality of points and lines in the plane
- Line sweep
- The art gallery problem
- Fortune's sweep-line Delaunay triangulation algorithm
- Quadtrees of points in the plane

Prerequisites

- CS 170 (Advanced Algorithms) or the equivalent.
- A basic course in probability.

Grading

- **80%** for the homeworks.
- **20%** for the project: a Delaunay triangulation implementation, or an alternative by arrangement with the instructor.

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