# Computational Geometry 202-2-5121 Fall 2015

Prerequisite: Algorithms

#### **Announcements:**

Assignment no. 2 (see below) is due December 29, 2015.

Assignment no. 1 (see below) is due December 1, 2015.

#### Instructor:

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Office hours: Sunday 12:00-14:00, Alon building (37), room 212, Tel: (08) 6461628

### **Teaching Assistant:**

222

Office hours:

#### **Class Time:**

Monday 16-18 (building 90, room 125) Tuesday 16-18 (building 90, room 239)

# **Course Description:**

This is an introductory course to computational geometry and its applications. We will present data structures, algorithms and general techniques for solving geometric problems, such as convex hull computation, line segment intersection, orthogonal range searching, construction of Voronoi diagram and Delaunay triangulation, polygon triangulation, and linear programming. We will also present several applications of geometric (optimization) algorithms to problems in robotics, computer graphics, GIS (geographic information systems), communication networks, facility location, manufacturing, and VLSI systems design.

# **Bibliography:**

The main textbook of the course is

[dBCvKO] <u>Computational Geometry: Algorithms and Applications (3rd edition),</u> M. de Berg, O. Cheong, M. van Kreveld and M. Overmars, Springer-Verlag, 2008.

#### **Additional** textbooks

[BY] Algorithmic Geometry, J-D Boissonnat and M. Yvinec, Cambridge University Press, 1998.

[DO] *Discrete and Computational Geometry*, S. Devadoss and J. O'Rourke, Princeton University Press, 2011.

[E] Algorithms in Combinatorial Geometry, H. Edelsbrunner, Springer-Verlag, 1987.

[M] <u>Computational Geometry: An Introduction Through Randomized Algorithms</u>, K. Mulmuley, Prentice Hall, 1994.

[O] Computational Geometry in C (2nd edition), J. O'Rourke, Cambridge University Press, 1998.

[PS] <u>Computational Geometry: An Introduction (2nd edition)</u>, F. Preparata and M. Shamos, Springer-Verlag, 1988.

## **Assignments, Exam and Grades:**

The final grade will be determined by 3-5 homework assignments (4% each) and a final exam.

Many of the exercises in the HW assignments are taken from [dBCvKO].

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Assignment no. 1 (due December 1, 2015)
Assignment no. 2 (due December 29, 2015)
Assignment no. 3 (due January 19, 2016)
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Some old exams: <u>exam 2005 A</u>; <u>exam 2005 B</u>; <u>exam 2007 A</u>; <u>exam 2007 B</u>; <u>exam 2009 A</u>; <u>exam 2009 B</u>

## **Topics:**

The following list of topics is tentative.

The *convex hull* of a set of points in the plane (applications: computing the diameter and width of a point set).

An *output sensitive* algorithm for computing the intersection points formed by a set of line segments; the *plane sweep* technique.

A representation for *planar maps* (based on doubly-connected edge lists).

Computing the *overlay* of two planar maps; boolean operations on two polygons (union, intersection, and difference).

The art gallery theorem; introduction to polygon triangulation.

An O(n log n) polygon triangulation algorithm (partitioning a polygon into y-monotone pieces; triangulating a y-monotone polygon).

Orthogonal range searching.

Casting; transforming the problem of determining whether a polyhedron P with n faces is castable into n instances of the problem of finding a point in the intersection of n half-planes. Computing the intersection of n half planes in O(n log n) time.

*Linear programming* - introduction; A randomized incremental algorithm for linear programming in the plane.

Planar *point location*, vertical decomposition / trapezoidal map, a randomized incremental algorithm.

Nearest site queries, nearest site Voronoi diagram.

Triangulation of a set of points in the plane; the *Delaunay triangulation*.

Arrangement of lines; duality; computing the discrepancy of a set of points in the unit square.

Segment trees; computing the area of a set of n axis-parallel rectangles in O(n log n) time.

Hidden surface removal: problem definition, image space / object space, the z-buffer algorithm, depth order, the painter's algorithm. Output sensitive hidden surface removal algorithm for horizontal fat triangles.

Introduction to *geometric optimization* through *facility location* optimization and *wireless networks*.

# **Course summary:**

Below you will find, after each class, a brief summary of the topics covered in class. This should not be taken as a complete description of the course's content.

#### 26.10.15

Introduction

#### 27.10.15

The *convex hull* of a set of points in the plane (gift wrapping, quickhull, O(n log n)-time incremental algorithm and divide and conquer algorithm

#### 2.11.15

The diameter and width of a set of points; rotating calipers

#### 3.11.15

Sweeping; an output sensitive algorithm for line segment intersection

#### 9.11.15

Sweeping with a ray;

Implementing the decision problem "do two line segments intersect?" and a more general discussion on implementation issues and general position

#### 16.11.11

The doubly-connected edge list representation for planar maps

#### 21.11.11

Map overlay; Boolean operations

# 10.11.15

The art gallery theorem, introduction to guarding problems

## 16.11.15

Introduction to polygon triangulation

## 17.11.15

Partitioning a polygon into y-monotone pieces

## 23.11.15

triangulating a y-monotone polygon

## 24.11.15

Orthogonal range searching

Last update: November 29, 2015