Home
• How to contact us
• How to apply
• Entrance requirements
• Management

The MPRI Programme
• Organisation
• Level 1 modules
• Level 2 modules

Practical information
• Calendar and timetable
• Internships
• Scholarships
• PhD grants

Algorithmique et combinatoire des graphes géométriques / Algorithms and combinatorics for geometric graphs (24h)

Course 2-38-1, year 2017-2018.

Teachers: Éric Colin de Verdière [http://www.di.ens.fr/~colin/] (CNRS & Université Paris Est Marne-la-Vallée) and Vincent Pilaud [http://www.lix.polytechnique.fr/~pilaud/] (teacher in charge of the course, CNRS & École Polytechnique).

What's new?

The course starts on Thursday September 14, from 12:45 to 15:45 in room 2035 of the Sophie Germain building.

The first exercice sheet [http://monge.univ-mlv.fr/~colinde/cours/17mpri/dm1.pdf] (for ÉCdV's course) is available and due on October 5.

The second exercice sheet [http://www.lix.polytechnique.fr/~pilaud/enseignement/MPRI/devoirMaisonMPRI17.pdf] (for VP's course) is available and due on October 26.

The second exercice sheet [http://monge.univ-mlv.fr/~colinde/cours/17mpri/dm2.pdf] (for ÉCdV's course) is available and due on November 9.

The final exam will be on November 23, usual time (12:45-16:45) and usual room (Sophie Germain building, room 2035). All documents and manuscript notes from the course are allowed. All other documents or electronic devices are forbidden. Plan to prepare two separated sheets for the two parts of the exam.

Practical information

When? First period, Thursday, 12:45-15:45. The tentative schedule is as follows:

- Sept 14. VP (planar graphs, crossing lemma) + ÉCdV (basic properties of planar graphs)
- Sept 21. ÉCdV (planarity testing, graph drawing, minimum spanning tree for planar graphs)
- Sept 28. VP (introduction to Schnyder woods) + ÉCdV (more algorithms for planar graphs: coloring, min-cut)
- Oct 5. VP (applications of Schnyder woods + polytopes)
- Oct 12. ÉCdV (classification of surfaces, shortest loops with cut locus)
- Oct 19. VP (polytopes + triangulations + secondary polytope)
- Oct 26. ÉCdV (shortest cut graph, homotopy and universal cover, and if time: shortening curves or min-cut on surfaces)
- Nov 2. NO LECTURE.
- Nov 9. VP (permutahedra and associahedra)
- Nov 23. Exam

Note that this tentative schedule is subject to modifications.

Where? Sophie Germain building, room 2035.

Language. Lessons will be given in French by default, or in English upon request of at least three persons who do not understand French, and if nobody objects. Lecture notes will be available in English. The exam will be translated if needed.

Evaluation. The evaluation is done with the final exam. Three exercises sheets will be proposed during the course period; if solved, they will give some extra credit for the final grade. The tentative schedule of the exercice sheets is as follows:

- Sept 21. Short exercice sheet for ÉCdV's course [http://monge.univ-mlv.fr/~colinde/cours/17mpri/dm1.pdf], to return on Oct 5.
- Oct 5. Long exercice sheet for VP's course [http://www.lix.polytechnique.fr/~pilaud/enseignement/MPRI/devoirMaisonMPRI17.pdf], to return on Oct 26.
- Oct 26. Short exercice sheet for ÉCdV's course [http://monge.univ-mlv.fr/~colinde/cours/17mpri/dm2.pdf], to return on Nov 9.
- Nov 23. Final exam (usual time, usual room). All documents and manuscript notes from the course are allowed. All other documents or electronic devices are forbidden. Plan to prepare two separated sheets for the two parts of the exam.

Prerequisites. None.

Main theme

Algorithms and combinatorics for graphs are a major theme in computer science. In this course, we study various aspects of this theme in the case of graphs arising in geometric settings. Examples include planar graphs (of course), graphs drawn without crossings on topological surfaces, and graphs of polytopes and other combinatorial structures. The course is therefore at the frontier of graph algorithms, combinatorics, and computational geometry.

Following this course is a good opportunity

- to see some relatively standard tools in algorithms and combinatorics applied in geometric settings,
- leading to results at the edge of current research, and
- to learn about some fundamental objects such as polytopes and surfaces, which appear in various contexts (optimization, discrete mathematics, topological graph theory).

Preliminary roadmap

- Graphs drawn in the plane (ÉCV + VP):
  - basics: combinatorial representations of planar graphs, topology, duality, Euler's formula;
  - planarity test and graph drawing algorithms;
  - crossing numbers of graphs;
  - efficient exact algorithms for planar graphs: minimum spanning tree, vertex coloring, minimum (s,t)-cut.
- Polytopes and geometric graphs (VP):
  - basics: polytopes and simplicial complexes, examples and properties;
  - planar triangulations, flips, and Delaunay triangulation;
  - more combinatorial structures: permutohedra and associahedra.
- Graphs on surfaces (ÉCV):
  - basics: classification theorem for surfaces up to homeomorphism;
  - topological algorithms for graphs on surfaces: shortest non-separating or non-contractible cycle, shortest system of loops;
  - depending on remaining time: testing homotopy with universal cover or efficient exact algorithm for computing a minimum (s,t)-cut or algorithm for shortening curves on surfaces.

Course notes

ÉCV's notes can be found here [http://www.di.ens.fr/~colin/cours/17mpri/poly.pdf]. VP's notes can be found here [http://www.lix.polytechnique.fr/~pilaud/enseignement/MPRI/notesCoursMPRI17.pdf].

Bibliography

- The course notes (since no book covers all these topics).
- Jesus A. De Loera, Jörg Rambau, and Francisco Santos. Triangulations: Structures for Algorithms and Applications, volume 25 of Algorithms and Computation in Mathematics. Springer Verlag, 2010.
- Stefan Felsner. Geometric graphs and arrangements. Advanced Lectures in Mathematics. Friedr. Vieweg & Sohn, Wiesbaden, 2004. Some chapters from combinatorial geometry.
- Bojan Mohar and Carsten Thomassen. Graphs on surfaces. Johns Hopkins Studies in the Mathematical Sciences. Johns Hopkins University Press, 2001.
- Günter M. Ziegler. Lectures on polytopes, volume 152 of Graduate Texts in Mathematics. Springer-Verlag, New York, 1995.

Relevant courses

The course has some connections with the following ones:

- 2-10 Algorithmic aspects of combinatorics;
- 2-14-1 Computational geometry learning;
- 2-29-1 Graph algorithms;
- 2-39 Computer Graphics and Scientific Visualization.

Additionally, this course fits well within a coherent list of courses on the theme “Algorithms and Complexity”, whose other courses are:

- 2-11-1 Advanced Algorithms;
- 2-11-2 Randomness in Complexity;
- 2-12-1 Techniques in Cryptography and Cryptanalysis;
- 2-13-2 Error correcting codes and applications to cryptography;
- 2-18-1 Distributed algorithms for the networks;
- 2-18-2 Algorithmique distribuée avec mémoire partagée;
- 2-24-1 Optimisation;
- 2-29-1 Graph algorithms;
- 2-33-1 Theory of Computations;
- 2-34-1 Quantum information and applications.